

ANNEXURE D: MCDM MODEL DESCRIPTION

OVERVIEW OF THE IDEAL MODE ANALYTICAL HIERARCHY PROCESS MULTICRITERIA DECISION-MAKING MODEL¹

Historical Development

The analytic hierarchy process (AHP) is a MCDM approach introduced by Saaty (1977), and is structured using sets of pairwise comparisons in a matrix to derive both the relative weights of the individual decision criterion (if required) and the rating of options in terms of each of the criteria. The pairwise comparison approach itself dates back to the eighteenth century and the mathematician and philosopher Marie Jean Antoine Nicolas Cariat, the Marquis de Condorcet, after which the Condorcet Method of voting using pairwise comparisons was named, is primarily credited for its development. The original AHP pairwise comparison model was later proven to be mathematically unstable by Belton and Gear (1983), based on the finding that it may influence the relative ranking of options with the introduction of an option that is similar or identical to one of the existing options. They then developed the ideal mode AHP as a variant of the original AHP, which proved to address this deficiency by adding an additional mathematical normalisation process to the calculation. This ideal mode AHP was later accepted by Saaty (1994) and according to Triantaphyllou and Mann (1995) is widely considered to be the most reliable MCDM methodology. It has increased in popularity amongst other MCDM tools and methodologies, mainly as a result of its simple mathematical structure and ease of use, typically in a matrix structure such as a spreadsheet.

Model Description

A technical methodological overview of the model is provided in subsequent paragraphs. When comparing options in a pairwise comparison using this model, as would be done for each of the criterion, the following scale of rating introduced by Saaty (1980) is used:

RATING SCALE TABLE	
Rating (R)	Description of Relative Rating
1	Equal preference
3	Weak preference
5	Essential or strong preference
7	Demonstrated preference
9	Absolute preference

When applying this scale, it is useful to first consider whether an option is better or worse than the option it is being compared to in respect of the criterion under consideration. This will then indicate whether the relative rating should be an integer value (when it is better) or a fraction (when it is worse), using the principle of reciprocal rating. The significance or severity of this preference is then expressed through the application of the numerical values in the scale, unless it is equal in which case a rating of 1 is used. Intermediate values (the equal numbers) could be used if required to indicate slight differences in rating. To create the pairwise comparison matrices, the first step would be to define the number of options and the number of criteria. To simplify the example, let us assume four options (A, B, C and D) and four criteria (C₁, C₂, C₃ and C₄), resulting in the following pairwise comparison matrix for the

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first criterion, where R_{1AB} represents the rating of option A compared to option B for criterion 1 (or answering the questions “is A better or worse than B?” and “what is the significance or severity of this preference?”):

OPTIONS MATRIX FOR CRITERION 1				
Options	A	B	C	D
A	R_{1AA}	R_{1AB}	R_{1AC}	R_{1AD}
B	R_{1BA}	R_{1BB}	R_{1BC}	R_{1BD}
C	R_{1CA}	R_{1CB}	R_{1CC}	R_{1CD}
D	R_{1DA}	R_{1DB}	R_{1DC}	R_{1DD}

Note that since $R_{1AA} = R_{1BB} = R_{1CC} = R_{1DD} = 1$ per definition as it represents the rating of an option compared to itself and $R_{1BA} = 1 / R_{1AB}$ etc. per definition as the one is the reciprocal of the other, only the cells indicated in bold italics in the top half of the matrix need to be rated. For the chosen example, similar matrices would be created for criteria C₂, C₃ and C₄. The same methodology could be used to determine the relative weighting of the criteria in relation to each other (P_{C1}, P_{C2}, P_{C3} and P_{C4}) that would later be applied to arrive at the overall ranking of options. To calculate the relative priorities, the geometric mean is first calculated per row, as shown below, where M_{1A} represents the geometric mean of rating results of option A for criterion 1, where 4 options are evaluated:

$$M_{1A} = (1 \times R_{1AB} \times R_{1AC} \times R_{1AD})^{1/4}$$

The relative priority of each option is then calculated per criterion by normalising the values, with the resultant formula for the calculation of the relative priority of option A for criterion 1 (P_{1A}) for the given example:

$$P_{1A} = M_{1A} / (M_{1A} + M_{1B} + M_{1C} + M_{1D})$$

These steps are easily completed by adding columns to the options matrix and the Original AHP decision matrix is then produced by copying the respective priority vector columns from the options priority matrices into a single matrix, with the criterion priorities from the criterion priority matrix in the top row. This matrix is then used to produce the ideal mode AHP decision matrix, by adjusting the relative options priority values through a second normalisation:

$$IP_{1A} = P_{1A} / (\text{maximum of } P_{1A}; P_{1B}; P_{1C}; P_{1D})$$

Similarly, these relative priority values are normalised for the other options and criterion, resulting in the ideal mode AHP decision matrix below, with the maximum IP value per criterion column having the value of 1:

IDEAL MODE AHP DECISION MATRIX				
Criteria	C ₁	C ₂	C ₃	C ₄
Priority	P _{C1}	P _{C2}	P _{C3}	P _{C4}
A	IP _{1A}	IP _{2A}	IP _{3A}	IP _{4A}
B	IP _{1B}	IP _{2B}	IP _{3B}	IP _{4B}
C	IP _{1C}	IP _{2C}	IP _{3C}	IP _{4C}
D	IP _{1D}	IP _{2D}	IP _{3D}	IP _{4D}

The final option priority is then calculated by using the formula below:

$$P_A = (IP_{1A} \times P_{C1}) + (IP_{2A} \times P_{C2}) + (IP_{3A} \times P_{C3}) + (IP_{4A} \times P_{C4})$$

As stated previously, these final option relative priorities are usually again normalised by dividing each through the total of all and is often also represented in a graph for ease of use. The numerical values of the results should not be interpreted directly, other than for the purposes of indicating relative importance.

In addition, this model allows for the testing or confirmation of the consistency of the rating through calculation of a consistency ratio (CR) and Saaty (1980) concluded that a CR of less than 0.10 (or 10%) is considered acceptable. To determine the consistency of ranking in any options matrix (or criterion priority matrix), the consistency index (CI) value is calculated first using the formula below for the example of 4 options, where E_{\max} denotes the approximation of the maximum eigenvalue:

$$CI = (E_{\max} - 4) / (4-1)$$

In this formula, the approximation of the maximum eigenvalue is calculated by adding each column in the priority matrix and multiplying the resultant vector by the priority vector, as shown below, where R_1A represents the numerical total of the ratings in column A for criterion 1, and then multiplying this resultant vector with the priority vector:

$$R_1A = R_{1AA} + R_{1BA} + R_{1CA} + R_{1DA}$$

A row could be added to the options matrix to indicate these totals. The calculation of the E_{\max} is then done by multiplication of the resultant vector of totals with the priority vector, using:

$$E_{\max} = (R_1A \times P_{1A}) + (R_1B \times P_{1B}) + (R_1C \times P_{1C}) + (R_1D \times P_{1D})$$

The consistency ratio (CR) is then calculated by dividing the CI value by the random consistency index (RCI) value, given in the table below for different numbers of options (Saaty (1980)):

RANDOM CONSISTENCY INDEX TABLE	
Number of Options	RCI
1	0
2	0
3	0.58
4	0.90
5	1.12
6	1.24
7	1.32
8	1.41
9	1.45

The consistency ratio (CR) is given by:

$$\boxed{\text{CR} = \text{CI} / \text{RCI}}$$

Resultant CR values higher than 0.10 (or 10%) warrants a re-evaluation of the pairwise comparisons in the particular matrix.

References

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- Triantaphyllou E and Mann S H (1995) *Using the Analytic Hierarchy Process for Decision Making in Engineering Applications: Some Challenges*. International Journal of Industrial Engineering: Applications and Practice, vol. 2, no. 1: pp 35 - 44.